AN APPROXIMATE METHOD OF COMPUTING HEAT TRANSFER IN GAS–ELECTRIC HEATERS

N. V. Diligenskii and L. I. Babenkova

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An approximate method of analytical computation of heat transfer in gas–electric heaters is proposed. The results of analytical computation using the obtained formulas are compared with the results of numerical solution and electrical simulation.

We consider the problem of heating of air during its forced flow in a tube carrying electric current.

In the formulation of the problem we assume that the process of heating occurs without heat exchange with the surrounding medium, the temperature drop along the thickness of the wall is small and the return flow of heat along the tube due to small temperature gradients along the length can be neglected.

Under these assumption the investigation of heat balance of a heater leads to the formulation of the problem in the form of the following system:

$$\frac{\partial \vartheta}{\partial \mathrm{Fo}} + \mathrm{Pe}\frac{\partial \vartheta}{\partial \xi} + A(\vartheta - \theta) = \mathrm{Pe}\,\vartheta_0(\mathrm{Fo})\,\delta(\xi), \tag{1}$$

$$C\frac{\partial \theta}{\partial \mathrm{Fo}} + A(\theta - \vartheta) = Q + D\theta_0(\xi)\,\delta(\mathrm{Fo}), \tag{2}$$

where

$$\vartheta = \frac{T_g - T_m}{T_M - T_m};\quad \theta = \frac{T_t - T_m}{T_M - T_m};\quad \xi = \frac{x}{l};\quad A = \frac{\alpha \Pi l^2}{S\lambda_g};$$

$$C = \frac{c_t\gamma_t\delta\Pi}{c_{Pg}\gamma_g S};\quad Q = \frac{\mathrm{Po}\,\lambda_t\delta\Pi}{\lambda_g S};\quad D = \frac{\lambda_t\delta\Pi}{\lambda_g S}.$$

A complete solution of system (1)-(2) is very laborious and unsuitable for application and therefore we do not give this solution here. Modern electrical heating processes are characterized by high values of the velocity of the operating substance; therefore it is advisable to make use of this fact for simplifying the mathematical models of heat transfer. Thus, for describing the heat transfer process in a preheater at high speeds of the operating substance A. L. Iskra proposed a simpler mathematical model [1]. However her solution does not have a very clear analytical form and requires numerical computations on a computer. Experience in using the tables proposed in [1] showed that they do not offer the possibility of computing the heat transfer of the initial segments of the heater and the initial instants of rapid transitional regimes.

We construct another approximate model of the heat transfer process for the case of high gas velocities which are of greatest interest in practical computations.

System (1)-(2) can be written in the following matrix form introducing the initial and boundary conditions into the equations:

$$\begin{bmatrix} \frac{\partial}{\partial \mathrm{Fo}} + \mathrm{Pe}\frac{\partial}{\partial \xi} + A & -A \\ -A & C\frac{\partial}{\partial \mathrm{Fo}} + A \end{bmatrix} \cdot \begin{bmatrix} \vartheta \\ \theta \end{bmatrix} = \begin{bmatrix} \mathrm{Pe}\,\vartheta_0(\mathrm{Fo})\,\delta(\xi) \\ Q + D\theta_0(\xi)\,\delta(\mathrm{Fo}) \end{bmatrix}. \tag{3}$$

We put system (1)-(2) in a form which can be solved for each of the variables ϑ and θ:

$$L\vartheta = E_1,\qquad L\theta = E_2. \tag{4}$$

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The determinant of the system is

$$L=\begin{vmatrix} \dfrac{\partial}{\partial \mathrm{Fo}}+\mathrm{Pe}\dfrac{\partial}{\partial \xi}+A & -A \\ -A & C\dfrac{\partial}{\partial \mathrm{Fo}}+A \end{vmatrix}$$

$$=C\frac{\partial^2}{\partial \mathrm{Fo}^2}+C\,\mathrm{Pe}\frac{\partial^2}{\partial \xi \partial \mathrm{Fo}}+AC\frac{\partial}{\partial \mathrm{Fo}}+A\frac{\partial}{\partial \mathrm{Fo}}+A\,\mathrm{Pe}\frac{\partial}{\partial \xi}.$$

The right hand sides E_1 and E_2 are respectively computed from the formulas

$$E_1=\begin{vmatrix} \mathrm{Pe}\,\vartheta_0(\mathrm{Fo})\,\delta(\xi) & -A \\ Q+D\theta_0(\xi)\,\delta(\mathrm{Fo}) & C\dfrac{\partial}{\partial \mathrm{Fo}}+A \end{vmatrix}$$

$$=\mathrm{Pe}\,\vartheta_0(\mathrm{Fo})\,\delta(\xi)\left(C\frac{\partial}{\partial \mathrm{Fo}}+A\right)+A\,[Q+D\theta_0(\xi)\,\delta(\mathrm{Fo})],$$

$$E_2=\begin{vmatrix} \dfrac{\partial}{\partial \mathrm{Fo}}+\mathrm{Pe}\dfrac{\partial}{\partial \xi}+A & \mathrm{Pe}\,\vartheta_0(\mathrm{Fo})\,\delta(\xi) \\ -A & Q+D\theta_0(\xi)\,\delta(\mathrm{Fo}) \end{vmatrix}=Q\left(\frac{\partial}{\partial \mathrm{Fo}}+\mathrm{Pe}\frac{\partial}{\partial \xi}+A\right)$$

$$+D\theta_0(\xi)\,\delta(\mathrm{Fo})\left(\frac{\partial}{\partial \mathrm{Fo}}+\mathrm{Pe}\frac{\partial}{\partial \xi}+A\right)+A\,\mathrm{Pe}\,\vartheta_0(\mathrm{Fo})\,\delta(\xi).$$

In [2, 3] definitions of asymptotic forms of functionals have been introduced depending on parameters using which one can construct different approximate descriptions for differential operators. By definition we shall assume a linear differential operator with constant coefficients depending on the numerical parameters ν, $N_\nu(D)$ (D is differentiation operator) the asymptotic form of operator $L_\nu(D)$ for $\nu \to \nu_0$:

$$L_\nu(D)\sim N_\nu(D), \quad \nu\to\nu_0,$$

if the functional

$$\int\limits_{R^n}\frac{\varphi(\omega)\,d\omega}{L_\nu(i\omega)}\sim\int\limits_{R^n}\frac{\varphi(\omega)\,d\omega}{N_\nu(i\omega)}, \quad \nu\to\nu_0, \tag{5}$$

where $\varphi(\omega)$ is a function from some basic function space.

In (5) the equivalence relation can be taken in the weak sense of [3] as well strong [2] convergence. On the basis of this representation it can be shown that the following representation is valid for our problem:

$$C\frac{\partial^2}{\partial \mathrm{Fo}^2}+C\,\mathrm{Pe}\frac{\partial^2}{\partial \mathrm{Fo}\,\partial \xi}+AC\frac{\partial}{\partial \mathrm{Fo}}+A\frac{\partial}{\partial \mathrm{Fo}}+A\,\mathrm{Pe}\frac{\partial}{\partial \xi}$$

$$\sim C\,\mathrm{Pe}\frac{\partial^2}{\partial \mathrm{Fo}\,\partial \xi}+A\,\mathrm{Pe}\frac{\partial}{\partial \xi};\quad \mathrm{Pe}\to\infty. \tag{6}$$

Introducing the asymptotic representation (6) into (4) we write the following approximate system for determining the solution:

$$C\,\mathrm{Pe}\frac{\partial^2\vartheta}{\partial \mathrm{Fo}\,\partial \xi}+A\,\mathrm{Pe}\frac{\partial\vartheta}{\partial \xi}=C\,\mathrm{Pe}\,\vartheta_0(\mathrm{Fo})\,\delta(\xi)\frac{\partial\vartheta}{\partial \mathrm{Fo}}$$

$$+A\,\mathrm{Pe}\,\vartheta_0(\mathrm{Fo})\,\delta(\xi)+A\,[Q+D\theta_0(\xi)\,\delta(\mathrm{Fo})]; \tag{7}$$

$$C\,\mathrm{Pe}\frac{\partial^2\theta}{\partial \mathrm{Fo}\,\partial \xi}+A\,\mathrm{Pe}\frac{\partial\theta}{\partial \xi}=Q\left(\frac{\partial\theta}{\partial \mathrm{Fo}}+\mathrm{Pe}\frac{\partial\theta}{\partial \xi}+A\right)$$

$$+D\theta_0(\xi)\,\delta(\mathrm{Fo})\left(\frac{\partial\theta}{\partial \mathrm{Fo}}+\mathrm{Pe}\frac{\partial\theta}{\partial \xi}+A\right)+A\,\mathrm{Pe}\,\vartheta_0(\mathrm{Fo})\,\delta(\xi). \tag{8}$$

We consider the problem with the right hand side and the boundary conditions determined in quadrature Fo, $\xi \geq 0$; we shall consider the solution also in this quadrature and seek this solution using Laplace transform. Transforming Eqs. (7)-(8) in Fo with parameters p and in ξ with parameter s, we obtain the following algebraic system:

$$\mathrm{Pe}\,s(Cp+A)\bar{\bar{\vartheta}}=(Cp+A)\,\mathrm{Pe}\,\vartheta_0+A(\bar{\bar{Q}}+D\bar{\theta}_0),$$

$$\mathrm{Pe}\,s(Cp+A)\bar{\bar{\theta}}=(p+\mathrm{Pe}\,s+A)(\bar{\bar{Q}}+D\theta_0)+A\,\mathrm{Pe}\,\vartheta_0,$$

from which we get

$$\bar{\bar{\vartheta}} = \frac{\bar{\vartheta}_0}{s} + \frac{A\bar{\bar{Q}}}{\mathrm{Pe}\, sC\left(p+\frac{A}{C}\right)} + \frac{AD\bar{\theta}_0}{\mathrm{Pe}\, sC\left(p+\frac{A}{C}\right)}; \tag{9}$$

$$\bar{\bar{\theta}} = \frac{p\bar{\bar{Q}}}{\mathrm{Pe}\, Cs\left(p+\frac{A}{C}\right)} + \frac{\bar{\bar{Q}}}{C\left(p+\frac{A}{C}\right)} + \frac{A\bar{\bar{Q}}}{\mathrm{Pe}\, sC\left(p+\frac{A}{C}\right)}$$

$$+ \frac{pD\bar{\theta}_0}{\mathrm{Pe}\, sC\left(p+\frac{A}{C}\right)} + \frac{D\bar{\theta}_0}{C\left(p+\frac{A}{C}\right)} + \frac{AD\bar{\theta}_0}{\mathrm{Pe}\, sC\left(p+\frac{A}{C}\right)} + \frac{A\bar{\vartheta}_0}{sC\left(p+\frac{A}{C}\right)}. \tag{10}$$

Inverting (9)-(10) for Fo, $\xi \geq 0$ we obtain

$$\vartheta(\xi,\ \mathrm{Fo}) = \vartheta_0(\mathrm{Fo}) + \frac{A}{\mathrm{Pe}\, C} \exp\left(-\frac{A}{C}\mathrm{Fo}\right) \int_0^{\xi}\int_0^{\mathrm{Fo}} Q(\xi',\ \mathrm{Fo}') \exp\left(\frac{A}{C}\mathrm{Fo}'\right) d\,\mathrm{Fo}'\, d\xi' + \frac{AD}{\mathrm{Pe}\, C}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)\int_0^{\xi} \theta_0(\xi')d\xi', \tag{11}$$

$$\theta(\xi,\ \mathrm{Fo}) = \frac{1}{\mathrm{Pe}\, C}\left[\int_0^{\xi} Q(\xi',\ \mathrm{Fo})\, d\xi' - \frac{A}{C}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)\right.$$

$$\left.\times \int_0^{\xi}\int_0^{\mathrm{Fo}} Q(\xi',\ \mathrm{Fo}') \exp\left(\frac{A}{C}\mathrm{Fo}'\right) d\,\mathrm{Fo}'\, d\xi'\right] + \frac{1}{C}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)$$

$$\times \int_0^{\mathrm{Fo}} Q(\xi,\ \mathrm{Fo}') \exp\left(\frac{A}{C}\mathrm{Fo}'\right) d\,\mathrm{Fo}' + \frac{A}{\mathrm{Pe}\, C}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)$$

$$\times \int_0^{\xi}\int_0^{\mathrm{Fo}} Q(\xi',\ \mathrm{Fo}') \exp\left(\frac{A}{C}\mathrm{Fo}'\right) d\,\mathrm{Fo}'\, d\xi' - \frac{DA}{\mathrm{Pe}\, C^2}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)$$

$$\times \int_0^{\xi} \theta_0(\xi')\, d\xi' + \frac{D}{C}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)\theta_0(\xi) + \frac{AD}{\mathrm{Pe}\, C}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)$$

$$\times \int_0^{\xi} \theta_0(\xi')\, d\xi' + \frac{A}{C}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)\int_0^{\mathrm{Fo}} \vartheta_0(\mathrm{Fo}') \exp\left(\frac{A}{C}\mathrm{Fo}'\right) d\,\mathrm{Fo}'. \tag{12}$$

Integrals (11)-(12) can be evaluated in closed form for a sufficiently wide class of functions $Q(\xi,\ \mathrm{Fo})$, For each particular case with constant power of the heat source $Q = \mathrm{const}$ the solution has the form

$$\vartheta(\xi,\ \mathrm{Fo}) = \vartheta_0(\mathrm{Fo}) + \frac{Q}{\mathrm{Pe}}\xi\left[1 - \exp\left(-\frac{A}{C}\mathrm{Fo}\right)\right]$$

$$+ \frac{AD}{\mathrm{Pe}\, C}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)\int_0^{\xi} \theta_0(\xi')\, d\xi', \tag{13}$$

$$\theta(\xi,\ \mathrm{Fo}) = \frac{Q}{\mathrm{Pe}\, C}\xi \exp\left(-\frac{A}{C}\mathrm{Fo}\right) + \frac{Q}{A}\left[1 - \exp\left(-\frac{A}{C}\mathrm{Fo}\right)\right]$$

$$+ \frac{Q}{\mathrm{Pe}}\xi\left[1 - \exp\left(-\frac{A}{C}\mathrm{Fo}\right)\right] - \frac{AD}{\mathrm{Pe}\, C^2}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)\int_0^{\xi} \theta_0(\xi')\, d\xi'$$

$$+ \frac{D}{C}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)\theta_0(\xi) + \frac{AD}{\mathrm{Pe}\, C}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)\int_0^{\xi} \theta_0(\xi')\, d\xi'$$

$$+ \frac{A}{C}\exp\left(-\frac{A}{C}\mathrm{Fo}\right)\int_0^{\mathrm{Fo}} \vartheta_0(\mathrm{Fo}') \exp\left(\frac{A}{C}\mathrm{Fo}'\right) d\,\mathrm{Fo}'. \tag{14}$$

For homogeneous boundary conditions we have

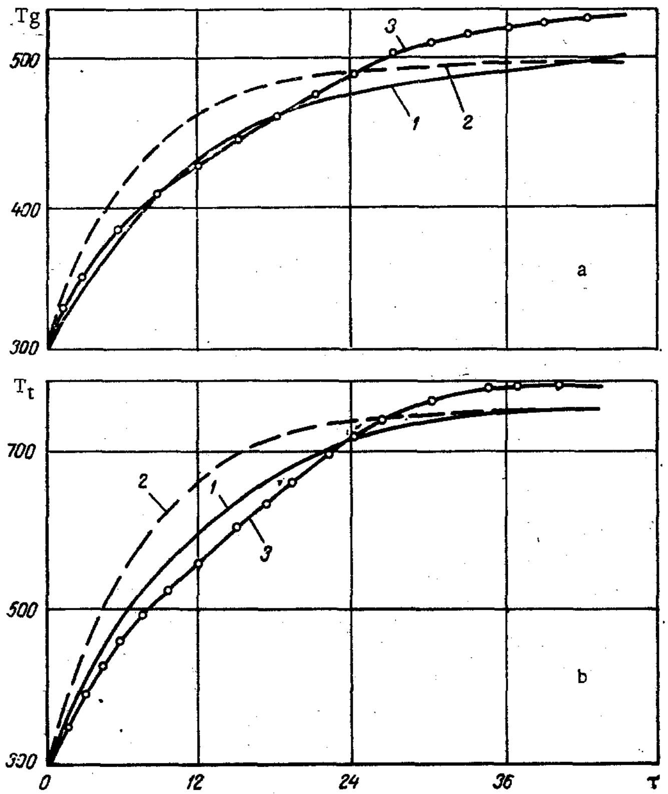

Fig. 1. Variation of the maximum temperature of gas (a) [1) computed from [1]; 2) from formula (15); 3) electrical simulation] (b) [1) computed from [1]; 2) from formula (16); 3) electrical simulation] in time. Tg, Tt in °K; τ in sec.

$$\vartheta(\xi, \mathrm{Fo}) = \frac{Q}{\mathrm{Pe}}\xi\left[1 - \exp\left(-\frac{A}{C}\mathrm{Fo}\right)\right], \tag{15}$$

$$\theta(\xi, \mathrm{Fo}) = \frac{Q}{\mathrm{Pe}\,C}\xi\exp\left(-\frac{A}{C}\mathrm{Fo}\right) + \frac{Q}{A}\left[1 - \exp\left(-\frac{A}{C}\mathrm{Fo}\right)\right] + \frac{Q}{\mathrm{Pe}}\xi\left[1 - \exp\left(-\frac{A}{C}\mathrm{Fo}\right)\right]. \tag{16}$$

The results of the computations using the analytical formulas (15)-(16) were compared with numerical computations of [1] for two real technological regimes (first regime: I = 200 A, V = 16 V, G = $10.2 \cdot 10^3$ kg/sec, w = 6.5 m/sec, P = $17.9 \cdot 10^5$ N/m², l = 1.2 m, d = 12 mm, δ = 0.5 mm, material of the tube ÉI437B; second regime: I = 274 A, V = 13.26 V, G = 105.10^3 kg/sec, w = 30.2 m/sec, P = $40 \cdot 10^5$ N/m², l = 1.2 m, d = 10 mm, δ = 1 mm, material of the tube Kh20N80T). The variation of the temperature of the gas and the tube with time is shown in Fig. 1a, b for the first variant.

An analysis of the computations shows that the agreement between the results of computations using formulas (15)-(16) and the tables of [1] improves as the gas velocity w increases. For a gas velocity of 30.2 m/sec the difference between the results is no more than 6%.

For a more objective estimate of the efficiency of the mathematical models an electrical simulation of the heat transfer process in the heater was done for the first regime. A more exact physical model was used in the simulation (real velocity profiles of the flow and turbulent transfer and radial and axial return flows of heat along the heater tubes were taken into consideration).

A comparison of the results of analytical computation and electrical simulation showed that for real heat transfer regimes computations using approximate formulas (13)-(14) and (15)-(16) are entirely admissible.

The proposed approach for high-speed heaters provides results with an accuracy that is adequate for engineering applications and greatly extends the range of problems permitting solution in a closed analytical form.

NOTATION

T	is the temperature
$Pe = wl/a_g$, $Fo = a_g\tau/l^2$, $Po = q_V l^2/\lambda_g(T_m - T_c)$	are the Peclet, Fourier, and Pomerantsov numbers;
x	is the spatial coordinate;
τ	is the time;
α	is the heat transfer coefficient;
a	is the thermal diffusivity;
C	is the specific heat;
λ	is the thermal conductivity;
q_V	is the specific density of heat generation;
w	is the velocity;
l, d, δ, π, s	are the length, inner diameter, thickness, perimeter, and area of the transverse cross section of the tube respectively;
$\delta(\xi)$, $\delta(Fo)$	are the Dirac's delta function;
T_m	is the scale temperature;
I	is the current intensity;
V	is the voltage;
G	is the gas flow rate;
P	is the gas pressure.

Subscripts

g	is the gas;
t	is the tube;
m	is the medium.

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